# Investments Problem Set 2 Lent 2024

**True-False (plus reasons; I am expecting no more than two-three sentences for each):**

1. **A** share that has a weekly volatility of returns of 5% will have an annual volatility of returns of about 35%
2. If share *A* is much more volatile than share *B*, then in the long term it is likely to do better than share *B*.
3. **If** share *A* and share *B* have a correlation of returns of 0.8, and if share *B* is very volatile then share *A*  must also be quite volatile.
4. **If** two shares have zero correlation, then it is possible to construct a portfolio consisting of the two shares that has zero volatility.
5. **If** two portfolios are both on the mean-variance efficient frontier, then the one that has the higher expected return will also have the higher volatility.
6. Assuming the CAPM is correct, a share with a higher beta will have a higher expected return than one with a lower beta even if it has much lower total risk.

**Questions**

1. If a share has a annual volatility of 25%, and a correlation with the market of 0.3, and the market has an annual volatility of 15%, what is the beta of the share, and what is its idiosyncratic risk?
2. In the previous question, if the risk-free rate is 4% and the market risk premium (the difference between the expected return on the market and the risk-free rate) is 6%, what is the expected return on the share?
3. **If** two shares each have a beta of 1 and annual volatility of 30% while the market has an annual volatility of 20%, what is the correlation between each of the shares and the market? What is the covariance of returns of the two shares, and what is the correlation between the two shares, assuming that the idiosyncratic risk of the two shares is uncorrelated?
4. How many shares would you need to hold in a portfolio to have an annual volatility of no more than 22%, assuming that they are as in the previous question (volatility of 30%, beta of 1 and idiosyncratic risk uncorrelated across stocks)?

**Problem**

The pension fund of the Die-cast and Bolt Corporation in the US is currently 100% invested in a diversified portfolio of US equities. At the last investment meeting, one of the trustees – an ex-investment banker - asked whether it really made sense to have all their bets on just one market. He said:

“The US market has done pretty badly recently, but I feel that it is still over-valued, and we could be heading for a further collapse. Surely it would be sensible to put some of the portfolio into non-US stocks. My own view is that the Japanese market, which has also under-performed recently as investors have fled equities generally, is now very under-valued by most yardsticks, and the economic fundamentals there are actually looking rather healthy. But I do not want to argue the case just on the grounds that Japan is a good bet. I also think we need to reduce our US exposure; Japan and the US have traditionally driven by very different factors, so it represents an excellent way of reducing risk without loss of expected return.”

The other trustees were not wholly convinced. The fund’s liabilities were almost entirely in the US and it made little sense to get exposed to the peculiarities of the Japanese economy. Far from reducing the fund’s risk, buying Japanese shares would actually increase the risk; not only would the fund be exposed to the Japanese economy, but also it would be affected by the exchange rate. If the yen went down against the dollar, the fund would suffer.

After much further discussion, they agreed to call for a report from consultants on the desirability of Japanese equity exposure. They also agreed guidelines for the report. The advice should assume that the US market had now become moderately attractive, with an expected return over the next year of 10% with a standard deviation of 16%. The Japanese market was probably a bit riskier with a standard deviation of 18%. But opinions were sharply divided about the expected return in Japan; to cope with this, the consultants were asked to consider two forecasts: a central forecast with an expected return of 10% (as in the US) and a pessimistic forecast of 6%. As for correlation, the consultants should assume that the historic figure of 0.35 for the correlation of returns between the US and Japan would be maintained.

* 1. Assuming that the trustees want to remain fully invested in equities, draw the set of feasible portfolios (that is all possible combinations of mean and standard deviation) if the fund is invested in a mix of US and Japanese equities for both forecasts. Is a 100% US portfolio efficient? If it is not efficient, identify a better portfolio that is efficient.
  2. The US interest rate is 3%. The fund is free to put some of its money into cash, when it will earn this rate. The fund can also borrow at the same rate. Draw the feasible set and again say whether a portfolio that is 100% invested in the US is efficient. If the portfolio is not efficient suggest a better portfolio that is efficient.
  3. The consultants’ report did suggest that the fund put some money into the Japanese market. This suggestion was met with scorn by one trustee, who argued as follows:

“The consultants are saying that even though the Japanese market is likely to do worse than the US, we should sell US equities and invest in Japan for what they call diversification reasons. They base their argument on this thing called “correlation”. They say that because the correlation of Japan with the US is only 0.35, we will reduce our risk by shifting into Japan. Frankly, I don’t much care about what they call risk – I am only interested in return. Can we really tell our members when they retire that while the bad news is that their pensions are pathetically small, the good news is that their pensions have small standard deviations?

But if you do take this risk stuff seriously, you still end up with complete nonsense. I took the trouble to look up the correlation between our own company’s stock (DBC) and the general US market. It too is only 0.35. So according to the consultants, we should buy it to reduce risk. I don’t object to this; as I have been arguing for years, our stock is greatly under-valued, and has been getting more so, so it represents a great buying opportunity – a darn sight better than Japan. But let’s not stop there – it turns out that there are very many other US stocks market that have a correlation with the general US market of around 0.35. Are our clever consultants saying that we should sell our US stocks and replace them with, wait for it, US stocks in order to reduce our risk? It sure improves the returns to our consultants and brokers, but I find it hard to see how our poor pensioners benefit.”

Is the argument valid? (Assume the numbers are correct).